Neutrino magnetic moment effects in electron-capture measurements at GSI

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Abstract

Oscillatory behavior of electron capture rates in the two-body decay $D \to R + \nu$ of hydrogen-like ion into recoil ion plus undetected neutrino ν, with a period of approximately 7 s, was reported in storage ring single-ion experiments at the GSI Laboratory, Darmstadt. Ivanov and Kienle [Phys. Rev. Lett. **103** (2009) 062502] have relegated this period to neutrino masses through neutrino mixing in the final state. New arguments are given here against this interpretation, while suggesting that these 'GSI Oscillations' may be related to neutrino spin precession in the static magnetic field of the storage ring. This scenario requires a Dirac neutrino magnetic moment μ_ν six times lower than the Borexino solar neutrino upper limit of $0.54 \times 10^{-10}\mu_B$ [Phys. Rev. Lett. **101** (2008) 091302], and its consequences are briefly explored.

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1 Introduction

Measurements of weak interaction decay of multiply ionized heavy ions coasting in the ion storage-cooler ring ESR at the GSI laboratory, since the first report in 1992 [1], open up new vistas for dedicated studies of weak interactions. In particular, electron capture (EC) decay rates in hydrogen-like and helium-like ^{140}Pr ions have been recently measured for the first time [2] by following the motion of the decay ions (D) and the recoil ions (R). The overall decay rates $\lambda_{\rm EC}$ of these two-body $^{140}{\rm Pr} \to {}^{140}{\rm Ce} + \nu$ EC decays, in which no neutrino ν is detected, are well understood within standard weak interaction calculations of the underlying $e^- p \to \nu_e n$ reaction [3,4]. EC decay rates reported subsequently in H-like and He-like ^{142}Pm ions [5] are consistent with these ^{140}Pr EC decay rate analyses. However, a time-resolved decay spectroscopy applied subsequently to the two-body EC decay of H-like ^{140}Pr and

^{142}Pm single ions revealed an oscillatory behavior, or more specifically a time modulation of the two-body EC decay rate [6]:

$$\lambda_{\rm EC}(t) = \lambda_{\rm EC}[1 + a_{\rm EC}\cos(\omega_{\rm EC}t + \phi_{\rm EC})], \tag{1}$$

with amplitude $a_{\rm EC} \approx 0.2$, and angular frequency $\omega^{\rm lab}_{\rm EC} \approx 0.89$ s^{-1} (period $T^{\rm lab}_{\rm EC} \approx 7.1$ s) in the laboratory system which is equivalent in the rest frame of the decay ion to a minute energy $\hbar\omega_{\rm EC} \approx 0.84 \times 10^{-15}$ eV. Subsequent experiments on EC decays of neutral atoms in solid environment have found no evidence for oscillations with periodicities of this order of magnitude [7,8]. Thus, the oscillations observed in the GSI experiment could have their origin in some characteristics of the H-like ions, produced and isolated in the ESR, and in the electromagnetic fields specific to the ESR which are not operative in normal laboratory experiments. Indeed, it is suggested here that the 'GSI Oscillations' could be due to the static magnetic field, perpendicular to the ESR, which stabilizes and navigates the motion of the ions in the ESR.

Several works, by Kienle and collaborators, relegated the 'GSI Oscillations' to interference between neutrino mass eigenstates that evolve coherently from the electron-neutrino ν_e [9,10,11,12,13,14]. This idea apparently also motivated the GSI experiment [6]. Such interferences, according to these works, lead to oscillatory behavior given by Eq. (1) with angular frequency ω_{ν_e} where, again in the decay-ion rest frame,

$$\hbar\omega_{\nu_e} = \frac{\Delta(m_\nu c^2)^2}{2M_D c^2} \approx 0.29 \times 10^{-15}\ {\rm eV}. \tag{2}$$

Here, $\Delta(m_\nu c^2)^2 = (0.76 \pm 0.02) \times 10^{-4}$ eV^2 is a neutrino squared-mass difference extracted from solar ν plus KamLAND reactor $\bar{\nu}$ data [15] for the two mass-eigenstate neutrinos that almost exhaust the coupling to ν_e, and $M_D \approx 130$ GeV/c^2 is the mass of the decay ion ^{140}Pr^{58+}. Although the value of $\hbar\omega_{\nu_e}$ on the r.h.s. of Eq. (2) is about three times smaller than the value of $\hbar\omega_{\rm EC}$ required to resolve the 'GSI Oscillation' puzzle, getting down to this order of magnitude nevertheless presents a remarkable achievement if correct. [1]

Other authors [18,19,20,21,22,23] have rejected any link between neutrino mass eigenstates and the EC decay rate oscillatory behavior reported by the GSI experiment [6], the underlying argument is that since no neutrino is singled out, the EC decay rate sums incoherently over amplitudes related to neutrino mass eigenstates, whereas any oscillatory behavior requires interference

[1] Eq. (2) was also obtained by Lipkin [16] assuming interference between two unspecified components of the initial state with different momenta and energies that can both decay into the same final state, an electron neutrino and a recoil ion with definite energy and momentum. This scenario was criticized by Peshkin [17].

between such amplitudes. To be more specific, if the time-dependent EC transition amplitude $A_{\nu_e}(i \to f;\ t)$, from initial state i (D injected at time $t = 0$) to a final state f (R plus a coherent combination of neutrino mass eigenstates at time t), is written in terms of transition amplitudes $A_{\nu_j}(i \to f;\ t)$ that involve mass-eigenstate neutrinos ν_j:

$$A_{\nu_e}(i \to f;\ t) = \sum_j U_{ej} A_{\nu_j}(i \to f;\ t), \tag{3}$$

where U_{ej} are mixing elements of the 3×3 (assumed unitary) matrix U

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U^*_{\alpha j}\, |\nu_j\rangle \qquad (\alpha = e, \mu, \tau) \tag{4}$$

between the emitted ν_e and mass-eigenstate neutrinos ν_j [24], then the associated probability is given by summing incoherently on $j = 1, 2, 3$:

$$\mathcal{P}_{\nu_e}(i \to f;\ t) = \sum_j |U_{ej}|^2 |A_{\nu_j}(i \to f;\ t)|^2 \approx |A_\nu(i \to f;\ t)|^2, \tag{5}$$

where the dependence of the absolute-squared terms $|A_{\nu_j}(i \to f;\ t)|^2$ on the species ν_j was neglected.[2] According to Eq. (5), the probability $\mathcal{P}_{\nu_e}(i \to f;\ t)$ for the two-body EC decay to occur is what standard weak interaction theory yields for a massless electron neutrino, regardless of its coupling to mass-eigenstate neutrinos. This holds true also for the total EC decay rate which is obtained by time differentiation of $\mathcal{P}_{\nu_e}(i \to f;\ t)$ plus integration over phase space and which is found identical with the time independent decay rate $\lambda_{\rm EC}$ derived ignoring neutrino mixing.

From the above discussion one notes that incoherence in terms of neutrino mass eigenstates rules out expressing the probability $\mathcal{P}_{\nu_e}$ as a squared absolute value of the amplitude A_{ν_e}:

$$\mathcal{P}_{\nu_e}(i \to f;\ t) \neq |A_{\nu_e}(i \to f;\ t)|^2. \tag{6}$$

It is instructive to ask whether incoherence shows up also in the flavor basis, since for times of order seconds which are appropriate to the 'GSI Oscillations' the coherence implied by Eq. (3) is still in effect and the flavor basis is of physical significance [22]. To this end I project Eq. (3) onto flavor β:

$$A_{\nu_e \to \nu_\beta}(i \to f;\ t) = \sum_j U_{ej} A_{\nu_j}(i \to f;\ t) U^*_{\beta j}, \qquad (\beta = e, \mu, \tau) \tag{7}$$

[2] This neglect does not hold for interference terms $A_{\nu_j} A^*_{\nu_{j'}}$, $j \neq j'$, which give rise to oscillatory behavior, as discussed in Sect. 2.

in close analogy with the discussion of neutrino flavor oscillations in dedicated oscillation experiments (Eq. (13.4) in Ref. [24]). The summed probability to have any of these three flavors appear in the final state, without specifying which one, is obtained by squaring $|A_{\nu_e \to \nu_\beta}(i \to f;\ t)|$ and summing over β:

$$\sum_\beta |\sum_j U_{ej} A_{\nu_j}(i \to f;\ t) U^*_{\beta j}|^2 = \sum_j |U_{ej}|^2 |A_{\nu_j}(i \to f;\ t)|^2, \tag{8}$$

where the assumed unitarity of the mixing matrix U, $\sum_\beta U^*_{\beta j} U_{\beta j'} = \delta_{jj'}$, was instrumental in eliminating the interference terms, leading to an incoherent sum identical with $\mathcal{P}_{\nu_e}(i \to f;\ t)$ of Eq. (5).

The purpose of the present paper is twofold. First, to show that even if the arguments given above against coherence are disregarded, and one chooses to evaluate $|A_{\nu_e}(i \to f;\ t)|^2$ as was done by Ivanov and Kienle in Ref. [13] contradicting Eq. (6) here, the resulting oscillation period would be many orders of magnitude shorter than required to explain the 'GSI Oscillations', and hence unobservable. More specifically, it is shown in Sect. 2 that the energy scale resulting by following the methodology of Ref. [13] is given by

$$\hbar\Omega_{\nu_e} = \frac{\Delta(m_\nu c^2)^2}{2E_\nu} \approx 0.95 \times 10^{-11}\ \text{eV}, \tag{9}$$

where $E_\nu \approx 4$ MeV is a representative value for neutrino energy in the H-like $^{140}\text{Pr} \to {}^{140}\text{Ce} + \nu_e$ and $^{142}\text{Pm} \to {}^{142}\text{Nd} + \nu_e$ EC decays [6]. The energy $\hbar\Omega_{\nu_e}$ is larger by over four orders of magnitude than $\hbar\omega_{\text{EC}}$ or $\hbar\omega_{\nu_e}$ given by Eq. (2), and so it would lead to modulation period shorter by over four orders of magnitude than the 7 s period reported by the GSI experiment. Given a time measurement resolution of order 0.5 s [6], the effect of such oscillatory behavior would average out to zero.

The main purpose of the present paper, however, is to introduce a new energy scale $\hbar\omega_{\mu_\nu}$, essentially given by the product of the neutrino magnetic moment μ_ν (or rather its upper limit) and the static magnetic field B which is perpendicular to the ESR. It is argued in Sect. 3 that precession of the neutrino spin in this magnetic field induces interferences that might lead to oscillations of the required period, namely that $\hbar\omega_{\mu_\nu}$ is commensurate with $\hbar\omega_{\text{EC}}$. The arguments provided in Sect. 3 are rather schematic and, judging by the various referee reports which helped to shape the final form of this published version, may appear controversial to many experts in the Neutrino community. Nevertheless, as stated by the last referee "it will no doubt create further discussions and opposing views" that "might help in reaching the required consensus."

2 Interference and time modulation of two-body EC rates

Here I show that a correct application of of the formalism followed by Ivanov and Kienle [13], accepting it for the sake of argument, leads to oscillations with angular frequency $\hbar\Omega_{\nu_e}$, Eq. (9); not with angular frequency $\hbar\omega_{\nu_e}$, Eq. (2), as claimed in Ref. [13]. To this end, I use as closely as possible their specific time-dependent first-order perturbation theory amplitudes $A_{\nu_j}(i \to f;\ t)$:

$$A_{\nu_j}(i \to f;\ t) = -i \int_0^t \langle f(\vec{q})\nu_j(\vec{k}_j)|H_{e\nu_j}(\tau)|i(\vec{0}\,)\rangle d\tau, \tag{10}$$

with a weak-interaction Hamiltonian for the leptonic transition $e^- \to \nu_j$ given by

$$\mathcal{H}_{e\nu_j}(\tau) = \frac{G_F}{\sqrt{2}} V_{ud} \int d^3x [\bar{\psi}_n \gamma^\lambda (1 - g_A \gamma^5) \psi_p][\bar{\psi}_{\nu_j} \gamma_\lambda (1 - \gamma^5) \psi_{e^-}]. \tag{11}$$

Here, $x = (\tau, \vec{x}\,)$, G_F is the Fermi constant, V_{ud} is the CKM matrix element, g_A is the axial coupling constant, and with $\psi_n(x)$, $\psi_p(x)$, $\psi_{\nu_j}(x)$ and $\psi_{e^-}(x)$ denoting neutron, proton, mass-eigenstate neutrino ν_j and electron field operators, respectively. EC decays occur at any time τ within $[0, t]$, from time $t' = 0$ of injection of D into the ESR to time $t' = t$ of order seconds and longer at which the EC decay rate is evaluated. In the single-ion GSI experiment [6] the heavy ions revolve in the ESR with a period of order 10^{-6} s and their motion is monitored nondestructively once per revolution. The decay is defined experimentally by the *correlated* disappearance of D and appearance of R, but the appearance in the frequency spectrum is delayed by times of order 1 s needed to cool R. The order of magnitude of the experimental time resolution is similar, about 0.5 s, as reflected in the time intervals used to exhibit the experimental decay rates $\mathcal{R}(t)$ in Figs. 3,4,5 of Ref. [6]. The time-averaged decay rates determined in the ESR appear to agree with those measured elsewhere, e.g. for ^{142}Pm [7], and this consistency suggests that details of kinematics and motion of the heavy ions in the storage ring affect little the overall decay rates which are evaluated here in conventional time-dependent perturbation theory. Therefore, it is plausible to assume that the evolution of the final state in these single-ion EC measurements at GSI proceeds over times of order 1 s which is used here as a working hypothesis.

To obtain the time dependence of the amplitude $A_{\nu_j}(i \to f;\ t)$ (similarly structured to Eq. (6) of Ref. [13]), recall that the time dependence of the

integrand in Eq. (10) is given by $\exp(\mathrm{i}\Delta_j \tau)$ where[3]

$$\Delta_j(\vec{q}\,) = E_R(-\vec{q}\,) + E_j(\vec{q}\,) - M_D \tag{12}$$

with

$$E_R = \sqrt{M_R^2 + (-\vec{q}\,)^2}, \quad E_j = \sqrt{m_j^2 + \vec{q}^{\,2}} \tag{13}$$

for the recoil ion and neutrino ν_j energies, respectively, in the decay-ion rest frame. Integrating on this time dependence results in a standard time-dependent perturbation-theory energy-time dependence [25]

$$A_{\nu_j}(i \to f;\ t) \sim \frac{1 - \exp(\mathrm{i}\Delta_j t)}{\Delta_j}. \tag{14}$$

The EC decay rate $\mathcal{R}_{\nu_e}(i \to f;\ t)$ is obtained from the probability $\mathcal{P}_{\nu_e}(i \to f;\ t)$, Eq. (5), by differentiating: $\mathcal{R} = \partial_t \mathcal{P}$. Using Eq. (14) for the time dependence of $A_{\nu_j}(i \to f;\ t)$, one gets a nonoscillatory contribution to $\mathcal{R}_{\nu_e}$:

$$\mathcal{R}_{\nu_j} = \frac{d}{dt}|A_{\nu_j}(i \to f;\ t)|^2 \sim \frac{2\sin(\Delta_j t)}{\Delta_j} \to 2\pi\delta(\Delta_j), \tag{15}$$

where the last step requires a sufficiently long time t. The properly normalized contribution of these terms to $\mathcal{R}_{\nu_e}(i \to f;\ t)$ is given by

$$\sum_j \mathcal{R}_{\nu_j} = \lambda_{\mathrm{EC}} \sum_j |U_{ej}|^2 \delta(\Delta_j) \approx \lambda_{\mathrm{EC}}\delta(\Delta), \tag{16}$$

where the dependence of $\delta(\Delta_j)$ on the species j could be safely neglected. If $j' \neq j$ interference terms are considered, then their properly normalized contribution to $\mathcal{R}_{\nu_e}(i \to f;\ t)$, again for sufficiently long times, is given by

$$\lambda_{\mathrm{EC}} \sum_{j>j'} \mathrm{Re}(U_{ej}U^*_{ej'})[\delta(\Delta_j) + \delta(\Delta_{j'})]\cos[(\Delta_j - \Delta_{j'})t]. \tag{17}$$

The Dirac δ functions in Eqs. (16) and (17) take care of energy conservation and have to be integrated upon, instead of the more customary integration on the implied c.m. momentum $\vec{q}$ to obtain the EC decay rate. It is straightforward to integrate over Δ for the nonoscillatory terms which then yield as expected the rate λ_{EC} in Eq. (16). For the oscillatory terms it is impossible to satisfy both $\delta(\Delta_j)$ and $\delta(\Delta_{j'})$ *simultaneously* in Eq. (17), meaning that the

[3] From here on $\hbar = c = 1$ units are almost exclusively used.

phase $(\Delta_j - \Delta_{j'})t$ is once evaluated under the constraint $\Delta_j(\vec{q}) = 0$ and once under the constraint $\Delta_{j'}(\vec{q}) = 0$. On each occasion, using a generic notation k for the momentum implied by each one of the δ functions, one obtains to an excellent approximation

$$\Delta_j(k) - \Delta_{j'}(k) = E_j(k) - E_{j'}(k) = \hbar\Omega_{jj'}, \tag{18}$$

where $\Omega_{jj'}$ is related to Ω_{ν_e} of Eq. (9):

$$\hbar\Omega_{jj'} = \frac{m_j^2 - m_{j'}^2}{2E_\nu} \approx \hbar\Omega_{\nu_e}. \tag{19}$$

Ivanov and Kienle [13] overlooked this subtlety by using in Eq. (18) simultaneously *on energy shell* momentum values k_j and $k_{j'}$ implied by $\delta(\Delta_j)$ and $\delta(\Delta_{j'})$ respectively, and replacing $\Delta_j - \Delta_{j'}$ in the oscillatory terms of Eq. (17) by $E_j(k_j) - E_{j'}(k_{j'}) \approx \hbar\omega_{\nu_e}$, Eq. (2). A similar error was made by Kleinert and Kienle when evaluating Eq. (54) in Ref. [11].

The requirement of *sufficiently long times* for Eq. (17) to hold translates in the present case to requiring $t \gg \Omega_{\nu_e}^{-1} \sim 7 \times 10^{-5}$ s, which is comfortably satisfied given the experimental time resolution scale of ~ 0.5 s [6]. Furthermore, as already discussed in Sect. 1, oscillations with periodicities of order 10^{-4} s would average out to zero in the GSI experiments, even if conceptually allowed.

3 Magnetic field effects

The preceding discussion ignored a possible role of the electromagnetic fields surrounding the ESR for guidance and stabilization of the heavy-ion motion. The nuclei ^{140}Pr and ^{142}Pm in the GSI experiment [6] have spin-parity $I_i^\pi = 1^+$, and the electron-nucleus hyperfine interaction in the decay ion forms a doublet of levels $F_i^\pi = (\frac{1}{2}^+, \frac{3}{2}^+)$, the 'sterile' $\frac{3}{2}^+$ level lying about 1 eV above the 'active' $\frac{1}{2}^+$ g.s. from which EC occurs to a $F_f = \frac{1}{2}$ final state of a fully ionized recoil ion with spin-parity $I_f^\pi = 0^+$ plus a left-handed neutrino of spin $\frac{1}{2}$.[4] The lifetime of the $F_i^\pi = \frac{3}{2}^+$ excited level is of order 10^{-2} s, so that it de-excites sufficiently rapidly to the $F_i^\pi = \frac{1}{2}^+$ g.s. [2,4]. Periodic excitations of this 'sterile' state cannot explain the reported time dependence and intensity pattern [26]. The static magnetic field which is perpendicular to the ESR, $B = 1.19$ T for ^{140}Pr [27], gives rise to precession of the $F_i^\pi = \frac{1}{2}^+$ initial-state spin with angular frequency ω_i of order $\hbar\omega_i \sim \mu_B B \approx 0.7 \times 10^{-4}$ eV [28], where μ_B is the Bohr magneton. The corresponding time scale of order 10^{-11} s is substantially

[4] The subscript f in this section relates to both the recoil ion and the neutrino.

shorter than even the ESR revolution period $t_{\rm revol} \approx 0.5 \times 10^{-6}$ s, so any oscillation arising from this initial-state precession would average out to zero over 1 cm of the approximately 100 m long circumference. A nonstatic magnetic field could lead through its high harmonics to oscillations with the desired frequency between the magnetic substates of the $F_i^\pi = \frac{1}{2}^+$ g.s. [29], but the modulation amplitude $a_{\rm EC}$ expected for such harmonics is substantially below a 1% level, and hence negligible. Furthermore, the associated mixing between the two hyperfine levels $F_i^\pi = (\frac{1}{2}^+, \frac{3}{2}^+)$ is negligible. In conclusion, no initial-state coherence effects are expected from internal or external electromagnetic fields in the GSI experiment.

In the final configuration, interferences may arise from the precession of the neutrino spin in the static magnetic field of the ESR. [5] The corresponding angular frequency ω_{μ_ν} is given by $\hbar\omega_{\mu_\nu} = \mu_\nu \gamma B < 0.5 \times 10^{-14}$ eV in the decay ion rest frame, due to the neutrino anomalous magnetic moment μ_ν interacting with the static magnetic field B. Here, $\gamma = 1.43$ is the Lorentz factor relating the rest frame to the laboratory frame, and $\mu_\nu < 0.54 \times 10^{-10} \mu_B$ from the Borexino solar neutrino data [30]. Below I show explicitly how the total EC rate gets time-modulated with angular frequency ω_{μ_ν}. To agree with the reported GSI measurements, $\omega_{\mu_\nu} = \omega_{\rm EC}$, a value of the electron-neutrino magnetic moment $\mu_\nu \sim 0.9 \times 10^{-11} \mu_B$ is required which is six times lower than provided by the published Borexino solar neutrino upper limit [30].

3.1 Interference due to a Dirac neutrino magnetic moment

For definiteness I first assume that neutrinos are Dirac fermions with only diagonal magnetic moments $\mu_{jk} = \mu_j \delta_{jk}$, and that these diagonal moments are the same for all three species: $\mu_j = \mu_\nu$. The emitted electron-neutrino is a left-handed lepton. The amplitude for producing it right-handed, namely with a positive helicity is negligible, of order $m_\nu/E_\nu < 10^{-7}$ and thus may be safely ignored. A static magnetic field perpendicular to the ESR flips the neutrino spin. Each of the mass-eigenstate components of the emitted neutrino will then precess, with amplitude $\cos(\omega_{\mu_\nu}\tau)$ for the depleted left-handed components and with amplitude $\mathrm{i}\sin(\omega_{\mu_\nu}\tau)$ for the spin-flip right-handed components [31]. Both are legitimate neutrino final states which are summed upon *incoherently*. The summed probability is of course time independent: $\cos^2(\omega_{\mu_\nu}\tau) + \sin^2(\omega_{\mu_\nu}\tau) = 1$. However, the magnetic field dipoles of the storage ring do not cover its full circumference, except for about 35% of it [27]. This results in interference between the decay amplitude $A^0_{\nu_j}$, for events with no magnetic interaction, and the decay amplitude $A^{\rm m}_{\nu_j}$ for events undergoing

[5] In disagreement with Merle's recent claim "a splitting in the final state cannot explain the GSI oscillations" [22] which ignored electromagnetic effects.

magnetic interaction with depleted left-handed components:

$$A^0_{\nu_j} \sim -\mathrm{i}\int_0^t \exp(\mathrm{i}\Delta_j\tau)d\tau, \quad A^{\mathrm{m}}_{\nu_j} \sim -\mathrm{i}\int_0^t \exp(\mathrm{i}\Delta_j\tau)\cos(\omega_{\mu_\nu}\tau)d\tau, \tag{20}$$

using the same normalization as in Eq. (14) for any of the left-handed mass-eigenstate neutrinos. This expression for $A^{\mathrm{m}}_{\nu_j}$ represents physically the action of the magnetic field at time τ of the EC decay.[6] The related amplitude $A^{\mathrm{R}}_{\nu_j}$ for events undergoing magnetic interaction which have resulted in a right-handed neutrino is then given by:

$$A^{\mathrm{R}}_{\nu_j} \sim -\mathrm{i}\int_0^t \exp(\mathrm{i}\Delta_j\tau)\mathrm{i}\sin(\omega_{\mu_\nu}\tau)d\tau. \tag{21}$$

Repeating the same steps in going from amplitudes A_{ν_j}, Eq. (14), to decay rates $\mathcal{R}_{\nu_j}$, Eq. (15), and adopting the same normalization, the decay rates associated with each one of these three amplitudes are given by:

$$\mathcal{R}^0_{\nu_j} = \frac{d}{dt}|A^0_{\nu_j}|^2 \sim 2\pi\delta(\Delta_j), \tag{22}$$

$$\mathcal{R}^{\mathrm{m}}_{\nu_j} = \frac{d}{dt}|A^{\mathrm{m}}_{\nu_j}|^2 \sim \frac{\pi}{2}[\delta(\Delta_j+\omega_{\mu_\nu})+\delta(\Delta_j-\omega_{\mu_\nu})](1+\cos(2\omega_{\mu_\nu}t)), \tag{23}$$

$$\mathcal{R}^{\mathrm{R}}_{\nu_j} = \frac{d}{dt}|A^{\mathrm{R}}_{\nu_j}|^2 \sim \frac{\pi}{2}[\delta(\Delta_j+\omega_{\mu_\nu})+\delta(\Delta_j-\omega_{\mu_\nu})](1-\cos(2\omega_{\mu_\nu}t)). \tag{24}$$

Note that although the two latter expressions for rates associated with the magnetic interaction are time dependent, their sum is time independent as expected from summing incoherently over the two separate helicities. The only time dependence in this schematic model arises from interference of the two amplitudes $A^0_{\nu_j}$ and $A^{\mathrm{m}}_{\nu_j}$ for a left-handed neutrino. Incorporating this interference, the total EC decay rate corresponding to ν_j is given by

$$\begin{aligned}\mathcal{R}_{\nu_j} &= \frac{d}{dt}(|a_0A^0_{\nu_j}+a_{\mathrm{m}}A^{\mathrm{m}}_{\nu_j}|^2+|A^{\mathrm{R}}_{\nu_j}|^2)\\ &\sim |a_0|^2 2\pi\delta(\Delta_j)+|a_{\mathrm{m}}|^2\pi[\delta(\Delta_j+\omega_{\mu_\nu})+\delta(\Delta_j-\omega_{\mu_\nu})]\\ &+2\mathrm{Re}(a_0a^*_{\mathrm{m}})\frac{\pi}{2}[\delta(\Delta_j+\omega_{\mu_\nu})+2\delta(\Delta_j)+\delta(\Delta_j-\omega_{\mu_\nu})]\cos(\omega_{\mu_\nu}t)\\ &+2\mathrm{Im}(a_0a^*_{\mathrm{m}})\frac{\pi}{2}[\delta(\Delta_j+\omega_{\mu_\nu})-\delta(\Delta_j-\omega_{\mu_\nu})]\sin(\omega_{\mu_\nu}t),\end{aligned} \tag{25}$$

[6] See Ref. [32] for a different choice of $A^{\mathrm{m}}_{\nu_j}$ that yields, nevertheless, the same time modulation as given by Eq. (26) below.

where $|a_{\rm m}|^2 \sim 0.35$ and $|a_0|^2 \sim 0.65$, with unknown relative phase between the probability amplitudes $a_{\rm m}$ and a_0 for undergoing or not undergoing magnetic interaction, respectively. Working out the complete normalization of this expression, the final rate expression is given by

$$\mathcal{R}_{\nu_e} = \lambda_{\rm EC}[1 + 2{\rm Re}(a_0 a_{\rm m}^*) \cos(\omega_{\mu_\nu} t)], \tag{26}$$

showing explicitly a time modulation of the kind Eq. (1) reported by the GSI experiment [6]. It is beyond the present schematic model to explain the magnitude of the modulation amplitude $a_{\rm EC}$ and the phase shift $\phi_{\rm EC}$, except that $|a_{\rm EC}| < 1$. In particular, a more realistic calculation is required in order to study effects of departures from the idealized kinematics implicitly considered above by which *both* the recoil ion and the neutrino go forward with respect to the decay-ion instantaneous laboratory forward direction. Whereas this is an excellent approximation for the recoil-ion motion, it is less so for the neutrino. [7] Nevertheless, for a rest-frame isotropic distribution, it is estimated that neutrino forward angles in the laboratory dominate over backward angles by more than a factor five.

For distinct diagonal Dirac-neutrino magnetic moments, Eq. (26) gets generalized to

$$\mathcal{R}_{\nu_e} = \lambda_{\rm EC}[1 + 2{\rm Re}(a_0 a_{\rm m}^*) \sum_j |U_{ej}|^2 \cos(\omega_{\mu_j} t)], \tag{27}$$

resulting in a more involved pattern of modulation. Finally, for vanishing diagonal magnetic moments, and nonzero values of transition magnetic moments, the discussion proceeds identically to that for Majorana neutrinos in the next subsection.

3.2 Majorana neutrino magnetic moments

Majorana neutrinos can have no diagonal electromagnetic moments, but are allowed to have nonzero *transition* moments connecting different mass-eigenstate neutrinos, or different flavor neutrinos. A static magnetic field perpendicular to the storage ring will induce spin-flavor precession [33]. However, the magnetic interaction effect is masked in this case by neutrino mass differences, such that the amplitudes $\cos(\omega_{\mu_\nu}\tau)$ and $\sin(\omega_{\mu_\nu}\tau)$ in Eqs. (20) and (21) are replaced, to leading order in $\omega_{\mu_\nu}/\Omega_{\nu_e} << 1$, by

$$\cos(\omega_{\mu_\nu}\tau) \to \exp(-{\rm i}\Omega_{jj'}\tau), \quad \sin(\omega_{\mu_\nu}\tau) \to \frac{\omega_{\mu_{jj'}}}{\Omega_{jj'}} \sin(\Omega_{jj'}\tau), \tag{28}$$

[7] I owe this observation to Eli Friedman.

where $\hbar\omega_{\mu_{jj'}} = \mu_{jj'}\gamma B$, and $\Omega_{jj'}$ is defined by Eq. (18). The period of any oscillation that might be induced by these amplitudes is of order $\Omega_{\nu_e}^{-1} \sim 7 \times 10^{-5}$ s which is several orders of magnitude shorter than the time resolution scale of ~ 0.5 s in the GSI experiment [6]. Therefore, such oscillations will completely average out to zero over realistic detection periods.

4 Discussion and summary

In this work I have discussed several interference scenarios which might be of relevance to the issue of 'GSI Oscillations'. It was reaffirmed that interference terms between different propagating mass-eigenstate neutrino amplitudes in two-body EC reactions on nuclei do not arise when no particular neutrino is singled out. A cancellation of such interference terms occurs also within a flavor oriented discussion, requiring however that the neutrino mass-flavor mixing matrix U is unitary. Interference terms of this kind arise and give rise to oscillatory behavior of the EC decay rate, if and *only* if a particular neutrino flavor is singled out. It was shown here and in Ref. [32] that the relevant period of oscillations is $T \sim 4\pi E_\nu/\Delta(m_\nu^2)$ which for $E_\nu \approx 4$ MeV as in the GSI experiments [6], and for $\Delta(m_\nu^2) \approx 0.76 \times 10^{-4}$ eV^2 [15], assumes the value $T \sim 4.4 \times 10^{-4}$ s, shorter by over four orders of magnitude than the period reported in these experiments. The oscillation period cited here is in full agreement with the oscillation length tested in dedicated neutrino oscillation experiments, [8] provided the time t is identified with L/c where L is the distance traversed by the neutrino between its source and the detector. In particular, besides the $\Delta(m_\nu^2)$ neutrino input, it depends on the neutrino energy E_ν, not on the mass M_D of the decay ion.

On the positive side, I have proposed a possible explanation of the 'GSI Oscillations' puzzle connected with the magnetic field that guides the heavy-ion motion in the ESR, requiring a Dirac neutrino magnetic moment μ_ν about six times lower than the laboratory upper limit value from the Borexino Collaboration [30]. The underlying mechanism is the interference between EC decay amplitudes not affected by the static magnetic field of the ESR and EC decay amplitudes affected by this field which induces spin precession of the emitted neutrino. Each of the outgoing neutrinos, provided it is left-handed, has two *indistinguishable* 'paths' to go through the ESR once it is produced in the EC decay: one is to encounter the static perpendicular magnetic field of the ESR, the other is to miss it. This is precisely like in the two-slit experiment. Interference is unavoidable then and is recorded by the motion of the entangled recoil ion in the ESR long after the neutrino has fled away.

[8] Detailed expressions are given in Eqs. (20,21,22) of Ref. [32] where a more rigorous wave-packet treatment would be required to justify the transition from t to L [34].

The spin-precession interference mechanism does not work for Majorana neutrinos that may have only *transition* magnetic moments. The resulting spin-flavor precession is suppressed by neutrino mass differences, and it becomes impossible to relate then the GSI Oscillations puzzle to magnetic effects. It is not yet resolved experimentally whether neutrinos are Dirac or Majorana fermions, although the theoretical bias rests with Majorana fermions, in which case the present paper accomplished nothing towards providing a credible explanation of this puzzle.

For experimental verification, note that the time-modulation period $T_{\rm EC}^{\rm lab}$ is inversely proportional to B, so the effect proposed here may be checked by varying B, for example by varying $\beta = v/c$ for the coasting decay ions. For a fixed value of β, B depends on the charge-to-mass ratio of the decay ion which varies only to a few percent with the decay-ion mass M_D. Finally, the proposed effect is unique to two-body EC reactions, since three-body weak decays do not constrain the neutrino direction of motion with respect to the fixed direction of $\vec{B}$. Indeed, preliminary data on the three-body β^+ decay of ^{142}Pm indicate no time modulation of the β^+ decay rate, limiting its modulation amplitude to $a_{\beta^+} < 0.03(3)$ [35].

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